EFFECT OF FLOW VELOCITY ON CURRENT DISTRIBUTION IN MGD CHANNEL

A. G. Korsun and A. L. Levin

We study stationary-plane flow of a conducting gas across a magnetic field in a channel of constant section formed by electrodes of finite length and insulators. The Hall effect is accounted for. We assume that the electromagnetic forces are small and the linear approximation is used. It is shown that disturbances of the compressible gas-flow velocity can lead to the formation of closed currents in the channel and alter significantly the current density distribution on the electrodes. For sufficiently large values of the gas velocity and conductivity, i.e., for magnetic Reynolds numbers which are not small, the current is concentrated near the exit from the interelectrode space.

The problem of parameter distribution in a MGD channel with account for the electromagnetic, gasdynamic, thermal, and so on processes is very complex. In many cases the primary forces and energy sources in the flow are determined by the electromagnetic quantities. In this connection it is of interest to study the electromagnetic processes in the channel with maximal simplification of the gasdynamic and thermal parts of the problem. The question of the current and potential distribution in a channel with finite electrodes when the conductivity, Hall parameter, and electrode drop are constant, the velocity does not vary along the axis of the channel, and the magnetic Reynolds number is small, has been examined in [1, 2]. The distributions of the electromagnetic quantities are determined by the current outflow from the interelectrode space and the Hall effect. Subsequent studies were made of the effect of electrode drop [3], ion slip [4], and conducting gas flow through the electrodes [5].

1. We examine stationary-plane flow of a conducting gas in a channel $|X| < \infty$, $0 \leq Y \leq h$ formed by insulators and electrodes of length $2ah$ (Fig. 1). We assume that the following conditions are satisfied:

1) Gas viscosity and thermal conduction are not significant;

2) the external magnetic field is uniform and directed along the Z axis (Fig. 1), and there is no current along this axis, therefore the overall magnetic field is directed along Z and depends on X and Y;

3) the plasma is quasineutral, the electrode drops are constant along the length of each electrode;

4) in the Ohm's law equation the terms proportional to the gradients of the pressure and electron temperature, and the terms associated with ion slip, are not significant [6].

Under these conditions the flow of a sufficiently dense singly ionized plasma may be described by the system of equations

$$\mathbf{j} + H\frac{\sigma}{\alpha\rho}\mathbf{j}\times\mathbf{B} = \sigma(-\nabla\varphi + \mathbf{v}\times\mathbf{B}), \qquad R_m\mathbf{j} = \nabla\times\mathbf{B}$$
$$\rho(\mathbf{v}\nabla)\mathbf{v} = -\frac{\nabla(\rho T)}{\gamma M^2} + A^2R_m\mathbf{j}\times\mathbf{B}, \qquad \nabla\cdot(\rho\mathbf{v}) = 0 \tag{1.1}$$
$$\rho T\mathbf{v}\nabla\ln\frac{T}{\rho^{\gamma-1}} = A^2R_mM^2\gamma(\gamma-1)\frac{j^2}{\sigma}, \quad \sigma = \sigma(T,\rho), \quad \alpha = \alpha(T,\rho)$$

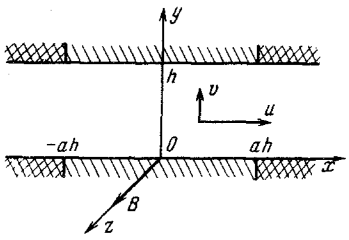

Fig. 1

All the quantities here are dimensionless. The scales for the density ρ, the two velocity components u and v (Fig. 1), magnetic field B, temperature T, conductivity σ, ionization ratio α, and the linear dimensions are respectively ρ_*, u_*, B_*, T_*, σ_*, α_*, and the channel width h. The scales for the current density j and the electric potential φ are

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$\sigma_* u_* B_*$ and $u_* B_* h_*$. The similarity criteria in (1.1) are: Mach number M, Alfven number A, Hall parameter H, magnetic Reynolds number R_m, and specific heat ratio γ:

$$M^2 = \frac{u_*^2}{\gamma R T_*}, \quad A^2 = \frac{B_*^2}{\mu \rho_* u_*^2}, \quad H = \frac{\sigma_* m_i B_*}{\alpha_* \rho_* e}, \quad R_m = \sigma_* u_* h \mu$$

Here R is the gas constant, μ the magnetic permeability, e and m_i the ion charge and mass.

Excluding j and φ from (1.1), it can be written for plane flow as

$$\begin{gathered}\frac{1}{\sigma}\Delta \mathbf{B} - \left(\nabla \frac{1}{\sigma}\right) \times (\nabla \times \mathbf{B}) + H\left(\nabla \frac{1}{\alpha\rho}\right) \times \nabla \frac{B^2}{2} = R_m\left[(\mathbf{v}\nabla)\mathbf{B} + \mathbf{B}(\nabla \mathbf{v})\right] \\ \rho(\mathbf{v}\nabla)\mathbf{v} = -\frac{\nabla(\rho T)}{\gamma M^2} - A^2 \nabla \frac{B^2}{2}, \qquad \Delta(\rho \mathbf{v}) = 0 \\ \rho T \mathbf{v} \nabla \ln \frac{T}{\rho^{\gamma-1}} = \frac{A^2 M^2}{R_m}\gamma(\gamma-1)\frac{(\nabla \times \mathbf{B})^2}{\sigma}, \qquad \sigma = \sigma(T, \rho), \quad \alpha = \alpha(T, \rho)\end{gathered} \tag{1.2}$$

If the changes of all the quantities in the channel are small in comparison with their average values, i.e., if in dimensionless form

$$\begin{gathered}|\rho - 1| \lesssim \varepsilon, \quad |u - 1| \lesssim \varepsilon, \quad |v| \lesssim \varepsilon, \quad |T - 1| \lesssim \varepsilon, \quad |B - 1| \lesssim \varepsilon, \\ |\sigma - 1| \lesssim \varepsilon, \quad |\alpha - 1| \lesssim \varepsilon\end{gathered}$$

where $\varepsilon \ll 1$, then Eq. (1.2) can be linearized. Then in the first equation, the induction equation, the nonlinear term with the Hall effect [if the parameter H is not large ($H \ll 1/\varepsilon$)] and the nonlinear term associated with the conductivity change are small in comparison with the first term. Similarly, in the energy equation for $M^2 A^2 R_m \ll 1/\varepsilon$ the term describing Joule dissipation is negligible.

After some transformations the system of linearized equations takes the form [7]

$$\begin{gathered}\Delta B = R_m \frac{M^2}{M^2 - 1}\frac{\partial v}{\partial y} + R_m \frac{M^2(1 - A^2) - 1}{M^2 - 1}\frac{\partial B}{\partial x} \\ \frac{\partial^2 v}{\partial x^2} - \frac{1}{M^2 - 1}\frac{\partial^2 v}{\partial y^2} = -\frac{A^2 M^2}{M^2 - 1}\frac{\partial^2 B}{\partial x \partial y} \\ \frac{\partial u}{\partial x} = \frac{1}{M^2 - 1}\frac{\partial v}{\partial y} - \frac{A^2 M^2}{M^2 - 1}\frac{\partial B}{\partial x} \\ \frac{\partial \rho}{\partial x} = -\frac{M^2}{M^2 - 1}\frac{\partial v}{\partial y} + \frac{A^2 M^2}{M^2 - 1}\frac{\partial B}{\partial x} \\ T = \rho^{\gamma - 1}\end{gathered} \tag{1.3}$$

At the walls which are impermeable to the gas, $v = 0$. The boundary conditions for B are obtained as follows. The second equation (1.1) permits writing the following relations for the dimensionless overall currents, referred to unit channel height along z, which flow through the lower (I_0) and upper (I_1) electrodes and along the channel (I_k)

$$\begin{gathered}R_m I_0 = B(-a, 0) - B(a, 0), \qquad R_m I_1 = B(-a, 1) - B(a, 1) \\ R_m I_k = B(x, 1) - B(x, 0)\end{gathered}$$

On ideal insulators the normal component of the current is zero, therefore on them $R_m j_y = -\partial B/\partial x$, i.e., $B = \text{const}$. In the following we examine conditions for which the overall current I_k along the channel between the insulating walls is zero, i.e., $B(x, 1) = B(x, 0)$ for $|x| \geq a$, therefore $I_0 = I_1 = I$. As the scale B_* for the magnetic field we use its value on the insulators to the left of the electrodes. If the current I in the external circuit is closed to the right of the electrodes, the B_* is the inductance of the external field, while if the current is closed to the left of the electrodes B_* is the sum of the inductance of the external field and the self-field of the current I. Thus the boundary conditions for B at the insulators have the form

$$\begin{gathered}B(x, 0) = B(x, 1) = 1 \quad \text{for} \quad x \leqslant -a \\ B(x, 0) = B(x, 1) = 1 - R_m I \quad \text{for} \quad x \geqslant a\end{gathered} \tag{1.4}$$

The magnitude of the current I can be found in terms of the difference φ_0 of the potentials between the electrodes, where $\varphi_0 > 1$ corresponds to the accelerator regime, $0 < \varphi_0 < 1$ corresponds to the generator regime, and $\varphi_0 < 0$ corresponds to the brake regime.

On ideally conducting electrodes the tangential component of the electric field intensity equals zero and the boundary condition for the overall magnetic field in the linear approximation is written as

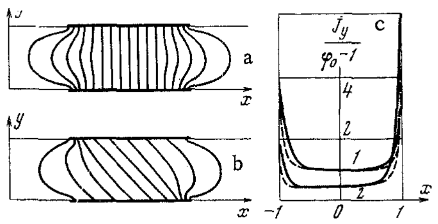

Fig. 2

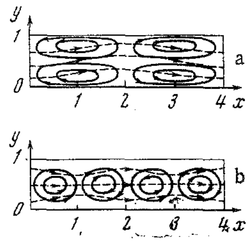

Fig. 3

$$\frac{\partial B}{\partial y} = H\frac{\partial B}{\partial x} \tag{1.5}$$

2. For further simplification of the problem we assume that the effect of the electromagnetic forces on the flow is small, i.e., in (1.3)

$$A^2M^2\partial B\ /\ \partial x \ll \partial v\ /\ \partial y$$

It follows from the solutions obtained below that this condition is satisfied for $A^2R_m \ll 1$. In this case the last four equations in (1.3) describe in the linear approximation compressible gas flow in the absence of electromagnetic forces. In the induction equation the first term on the right accounts for the influence of the varying compressible gas flow velocity on the distribution of the electromagnetic quantities. In an incompressible gas, i.e., for $M^2 \ll 1$, this term is negligibly small.

In supersonic flow the particular solutions of the gasdynamic equations of (1.3) have the form

$$\begin{gathered} v = v_0 \sin(r\pi y)\sin[k_r(x-x_0)],\ k_r = \frac{r\pi}{\sqrt{M^2-1}} \quad (r = 0, 1, 2, \ldots) \\ u = u(x_0, y) - \frac{v_0}{\sqrt{M^2-1}}\cos(r\pi y)\{\cos[kr(x-x_0)] - 1\} \\ \rho = 1 + \frac{v_0 M^2}{\sqrt{M^2-1}}\cos(r\pi y)\cos[k_r(x-x_0)] \end{gathered} \tag{2.1}$$

Here v_0 and x_0 are constants of integration.

This solution describes supersonic-adiabatic flow of an inviscid gas in a flat channel of constant section with small transverse displacements, which lead to periodic variations of the flow parameters along x with the period

$$L = 2r^{-1}\sqrt{M^2-1}$$

Disturbances of this type can arise if the flow at the channel entrance is nonuniform across the section under the action of transverse components of the electromagnetic forces associated with the currents flowing out of the interelectrode space [8].

3. To solve the induction equation we used the approximate method suggested for similar problems by Kalikhman, which is a version of the method of integral relations. The dependence of the overall B on the y coordinate was represented in the form of a fourth degree polynomial with coefficients which are functions of x

$$B = B_0 + B_2 y + B_3 y^2 + B_4 y^3 + (B_1 - B_0 - B_2 - B_3 - B_4)\, y^4 \tag{3.1}$$

Here B_0 and B_1 are the values of the induction at the channel walls for y = 0 and y = 1, respectively. The following five relations were used to find these five coefficients: the boundary conditions (1.4) or (1.5) at the two walls, the equation itself for y = 0 and y = 1, and the integral relation obtained by integrating the induction equation over y from 0 to 1 with the use of (3.1).

Thus, the solution of the partial differential equation with nonhomogeneous boundary conditions reduces to the solution of systems of ordinary differential equations for three regions: for the interelectrode space ($|x| \le a$) and the two channel segments with insulating walls ($|x| \ge a$). The constants of integration are determined by the finiteness of the $|x| \to \infty$ and the conditions for joining of the solutions at the boundaries of the regions.

Here we use the continuity at the boundary of the tangential component of the electric field intensity and the normal component of the current density. In the approximate solution the joining conditions reduce

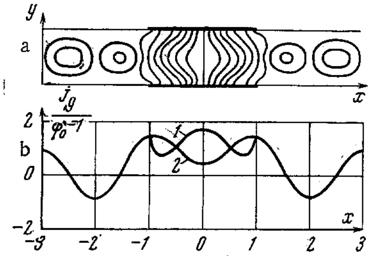

Fig. 4

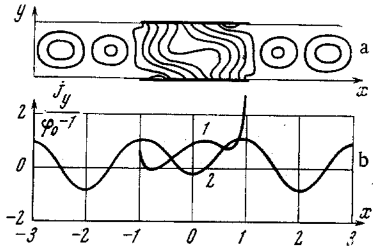

Fig. 5

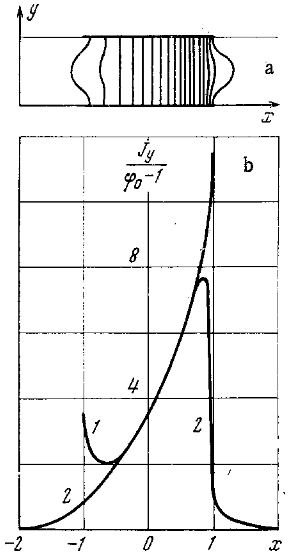

Fig. 6

to continuity of the difference of the potentials on the channel walls and the continuity of B at the walls and at the centerline of the channel. The value of I was found in terms of the potential difference φ_0 with the aid of the relation obtained from the linearized Ohm's law equation

$$\int_0^1 \frac{\partial B}{\partial x}\, dy = R_m\left[\int_0^1 B\,dy + \int_0^1 (u-1)\,dy - \varphi_0\right] - H(B_1 - B_0)$$

The problem was solved in [1, 2, 4] by different methods for $A^2 \ll 1$, $R_m \ll 1$, $v_0 = 0$. The solution obtained using the technique described above agrees approximately with these solutions. The coefficient characterizing the overall current increase owing to current outflow from the interelectrode space for $H \ll 1$, $R_m \ll 1$, $v_0 = 0$ equals $I/(\varphi_0 - 1) = 2a + 0.53$ rather than $I/(\varphi_0 - 1) = 2a + 0.44$ from the exact solution [1]. Figure 2 shows the electric current lines (i.e., the lines B = const) in a channel with finite electrodes for $a = 1$, $A^2 \ll 1$, $R_m \ll 1$, $v_0 = 0$, $H \ll 1$ (Fig. 2a) and $H = 1$ (Fig. 2b).

Figure 2c shows the current density distribution on the electrodes for $H \ll 1$ (curve 1) and for $H = 1$ (curve 2). The solid curves were obtained from the approximate solution and the dashed curves from [4]. The current density increases toward the end of the electrode, but not to infinity as in the exact solution, rather to a finite value.

4. In a channel with nonconducting walls disturbances of the type (2.1) lead to the formation of current loops. The corresponding solution of the induction equation for $R_m \ll 1$ has the form

$$B = 1 + V_r \sin[k_r(x - x_0)]\left\{{}^1/_2\,[y^2 - 2S_r y^3 + 2(S_r - 1)y^4] - \frac{k_r^2\,({}^1/_3 - {}^1/_4 S_r) + 5(S_r - 1)}{K_r^2 + n^2}\,(y - 2y^3 + y^4)\right\}, \qquad n^2 = 10 + {}^1/_4 R_m^2$$

$$V_r = \frac{R_m M^2 r \pi v_0}{M^2 - 1}, \qquad s_r = {}^2/_3 \;\text{ for }\; r = 1, \qquad s_\varphi = 1 \;\text{ for }\; r = 2$$

The electric current lines (solid) and the gas streamlines (dashed) for $M^2 = 5$, $R_m \ll 1$, $v_0 = 0.1$ are shown in Fig. 3a for r = 1 and in Fig. 3b for r = 2.

If a conducting gas stream travels with supersonic velocity in a channel with electrodes and if there are in the channel disturbances of the type (2.1), then current loops analogous to those shown in Fig. 3 are superimposed on the field of the currents flowing between the electrodes (Fig. 2). In the electrode region these loops close on the electrodes and alter the current-density distribution on the electrodes. The solution obtained on the induction equation for arbitrary H, R_m, v_0 is very complex.

Figure 4a shows the electric current lines in a channel with electrodes for $M^2 = 5$, $R_m \ll 1$, $v_0/(\varphi_0 - 1)$, r = 2, $a = 1$, $x_0 = -1$, $H \ll 1$. Figure 4b shows the distribution of the current density j_y at the electrodes (curve 1) and at the channel centerline (curve 2). For the same values of the parameters but with H = 1 the analogous distributions are shown in Fig. 5a, b (curve 1 is the distribution of j_y on the lower electrode; the distribution on the upper electrode is symmetric about the x = 0 axis). We see from the solution that for

$$\frac{v_0}{\varphi_0 - 1} \sim \frac{M^2 - 1}{M^2}\,\frac{k_r^2 + l^2}{2\pi k_r} \qquad (l^2 = 60 + R_m^2/4)$$

disturbances in the gas flow alter significantly the current distribution in the channel and on the electrodes.

Specifically, on some segments of the electrodes the current density may be very small. For sufficiently large values of the parameter $v_0/(\varphi_0-1)$ the solution shows the possibility of the appearance on the electrodes of segments with reversed direction of the current, i.e., current loops closing through the electrodes (Fig. 5a). Closure of the current lines on the electrode was discovered in a numerical solution of the problem of two-dimensional MGD flow in a coaxial system [9].

Flow nonuniformity alters the overall resistance of the interelectrode space. For $H \ll 1$, $R_m \ll 1$, and $r = 2$ the overall current is

$$I = \frac{1}{R_m}[B_0(-a) - B_0(a)] = (\varphi_0 - 1)\left(2a + \frac{5}{3n}\right) + \frac{V_r \cos(k_r x_0)}{3l^2(k_r^2+n^2)}[5nk_r \cos(k_r a) + (k_r^2 + l^2)\sin(k_r a)]$$

5. If R_m is small, then along with the effects examined above there appears a "deflection" of the electric current lines by the flow. For example, the solution for uniform flow for $v_0 = 0$, $H \ll 1$ has the form

$$B = 1 - \frac{5R_m(\varphi_0 - 1)}{n - {}^1/_2 R_m}\exp[(n + {}^1/_2 R_m)(a + x)] \quad (y - 2y^3 + y^4) \quad \text{for} \quad x \leqslant -a$$

$$B = B_0(x) + B_3(x)(y^2 - 2y^3 + y^4) \qquad \text{for} \quad |x| \leqslant a$$

$$B_0(x) = 1 + (\varphi_0 - 1)\{1 - \exp[R_m(a + x)]\} + \frac{2}{l^2}\{B_3(-a)\exp\,[R_m(a + x)] - B_3(x)\}$$

$$B_3(x) = -\frac{25R_m(\varphi_0 - 1)}{2\,\mathrm{sh}\,(2la)\,(n^2 - {}^1/_4 R_m^2)}\{[2n\,\mathrm{ch}\,(la + {}^1/_2 R_m a) + R_m\,\mathrm{sh}\,(la + {}^1/_2 R_m a)]\exp(-lx + {}^1/_2 R_m x)$$

$$- [2n\,\mathrm{ch}\,(la - {}^1/_2 R_m a) - R_m\,\mathrm{sh}\,(la - {}^1/_2 R_m a)]\exp(lx + {}^1/_2 R_m x)\}$$

$$B = B_0(a) + \frac{5R_m(\varphi_0 - 1)}{n + {}^1/_2 R}\exp[(n - {}^1/_2 R_m)(a - x) \quad (y - 2y^3 + y^4] \quad \text{for} \quad x \geqslant a$$

The electric current lines corresponding to this solution for $R_m = 1$ are shown in Fig. 6a and the current density j_y distributions are shown in Fig. 6b (curve 1 is at the electrodes, curve 2 is at the channel centerline).

For $2aR_m \leqslant 0.1$ the solution practically coincides with that for $R_m \ll 1$. The current concentration near the exit from the interelectrode space is associated with increase of

$$\left|\varphi_0 - \int_0^1 uB\,dy\right|$$

because of the approximately exponential variation of the overall B.

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